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Revelling in abstract maths

University of Auckland mathematician Eamonn O'Brien is making the most of his 2008 NZIMA Maclaurin Fellowship. He spoke with Jenny Rankine.

O'Brien describes himself as "a bit of a butterfly collector; a lot of my work has been on the development of good algorithms for the construction and classification of groups. I've developed techniques to count the number of groups of prime-power order; for example, we can count the groups of the order 1024, 2^{10}. The answer is about 50 billion and it involves a lot of computing."

During his fellowship, O'Brien completed the verification of the long-standing Ore conjecture on finite simple groups with Professor Martin Liebeck of Imperial College, London; Professor Aner Shalev of the Hebrew University of Jerusalem and Professor Pham Tiep at the University of Arizona.

The conjecture, posed in 1951, states that every element of every finite non-abelian simple group is a commutator. "Despite its elegance and simplicity, it has withstood many attacks," he says.

O'Brien also worked with a University of Auckland post-doctoral fellow, Henrik Bäärnhielm, on the development and implementation of Monte Carlo algorithms to construct a chief series for a linear group.

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results; they tend to be used when it is infeasible to compute an exact result with a deterministic algorithm. The chief series algorithm breaks groups into simple building blocks. Knowledge of such a series allows the use of many other algorithms to study the groups.

He and Professor Charles Leedham-Green, from Queen Mary University of London also worked on constructing short presentations for the classical groups of Lie type.

These groups form the central classes of non-abelian finite simple groups. "Such presentations are useful theoretically and practically, particularly in verifying the putative chief series for a linear group," he says.

With Mike Newman from the Australian National University, O'Brien studied the structure of odd order p-groups of fixed coclass.

These are groups that have a power of an odd prime number as their number of elements, with a fixed difference between their composition length and their smallest central number series. With Professor Bettina Eick at the Braunschweig University of Technology and Leedham-Green, they are attempting to understand periodicity among these p-groups, and how it can be used to describe infinite families of groups by a finite diagram or tree.

In May, O'Brien will give a series of lectures to graduate students in China, and in August he will give four invited lectures at the Groups St Andrews Meeting in Bath, the biggest international conference in group theory.

O'Brien describes himself as "very comfortable with abstraction". Despite this, many of his algorithms are part of the basic infrastructure of Magma, a computational algebra system. "People doing computation will frequently use algorithms I'm responsible for, and often for applications or areas I didn't have in mind."

The product replacement algorithm he developed with others in the 1990s has become a standard for mathematicians and statisticians wanting to choose an element reflective of certain properties in large finite groups.

See also

The Magma computational algebra system - http://magma.maths.usyd.edu.au/magma/

$$\left\langle \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right\rangle = G$$

$$|G| = 6$$