

## NOTABLE MATHS PROBLEMS

### POINCARÉ CONJECTURE

(1900) If  $M$  is a 3-manifold with trivial fundamental group, and  $\pi_i(M)=0$  for  $i=1,2$  and  $\pi_3(M)=\mathbb{Z}$  (ie,  $M$  has the homotopy groups of a 3-sphere), then  $M$  is homeomorphic to the 3-sphere.

**Simply:** (1904) That if any loop on the surface of a three-dimensional shape can be shrunk to a point (as any loop can on a 3-D sphere) then the shape is just a 3-D sphere.

**Discipline:** Topology

**Originator:** Jules Henri Poincaré, 1854-1912.

**Incentive:** \$US1million, one of the seven Millennium Prize Problems of the Clay Mathematics Institute.

**Notable false proof:** JHC Whitehead, 1934.

**Has led to:** Interesting new examples of 3-manifolds; several celebrated cases of Poincaritis.

**Unusual aspect:** Solving this problem in four and more dimensions has been much easier than solving it in three.

**Likely proof:** Grigori Perelman, Steklov Institute of Mathematics, St Petersburg, 2002 and 2003, although the Clay prize has yet to be awarded.

**NZIMA programme connection:** Geometric Methods in the Topology of 3-Dimensional Manifolds.



# Quandles

## knots & manifolds

**Topology is a pure maths discipline, but a focus of the NZIMA Summer Meeting on topology in 2006 was the cracking of current banking and financial encryptions. Jenny Rankine investigates.**

Meeting speakers Michael Freedman and Kevin Walker, of Microsoft, explored topological quantum field theory, developed originally to measure the set of all loops on a surface – what topologists call the fundamental group. They discussed the potential of these coding methods for breaking cryptographic systems based on integer factorisation, widely used in banking.

Co-director of the NZIMA programme on Geometric Methods in the Topology of 3-Dimensional Manifolds, Professor David Gauld, of the University of Auckland, said the

Taipa meeting this year attracted 60 people, from Australia, Japan, North America, the UK and around New Zealand.

The meetings were initiated by programme co-director Professor Vaughan Jones in 1994, and focus on a different branch of mathematics each year. Until this year Professor Jones was the southern hemisphere's only Fields Medallist, the maths equivalent of the Nobel Prize. It was awarded to him in 1990 by the International Mathematical Union for a new polynomial knot invariant (an object that distinguishes one theoretical knot from another), which was named after him.

Topology is sometimes called rubber sheet geometry, because it concerns itself with the spatial properties that are preserved after shapes are stretched or deformed without breaking. It does not distinguish between a square and a circle (as a rubber band circle can be stretched into a square) and it ignores distances (so that two different sized circles are equivalent in a topological universe).

Professor Gauld is studying manifolds – abstract mathematical spaces – that are too big to measure; some physicists think the universe is an example. "If we're trying to apply the maths to the universe, there's a long way to go," he says. "I'm still dabbling in two dimensions and physicists want solutions for three to ten dimensions."

The programme's third co-director, Dr Roger Fenn, of the University of Sussex, is working with PhD student Stevie Budden on quandles. These algebraic objects give rise to knot invariants. BSc honours student Michael Brough is doing his dissertation on a new knot invariant. Applications of knot theory include DNA recombination.

"People say pure mathematicians are just playing games with bunches of rules," says Gauld. "The amazing thing is that so often, ten or 50 years later, these great applications arise. When I first heard about topological quantum field theory in 1994, there was no mention of their connection with banking encryptions."



**Meeting participants Gabriela Slezakova and Qing Zhang, students at Waikato and Massey. Right: Vaughan Jones.**



$$\Delta = -t^{-3} + 3t^{-2} - 3t^{-1} + 4 - 4t + 3t^2 - 2t^3 + t^4$$