From IMAges 1, October 2006

America's Cup yachts are wrapped in mathematics. Jenny Rankine investigates.

More than the boats are battling

Emirates Team New Zealand NZL84 in a practice race against Desafio Espanol ESP88 in July 2006. Photo: Chris Cameron, ETNZ.

Two powerful yachts strain, leaping half out of the water with each wave, sails the size of 747 wings taut with a massive press of air. For most spectators, it is a duel between two crews. Sponsors' brands are hoping for a win for their boat. For the backroom teams, the two boats are the products of particular suites of mathematical models, proven or defeated in the media glare of the America's Cup.

NZIMA programme leader Professor Andy Philpott, from the University of Auckland Engineering Science Department, has been involved in every America's Cup since he developed a velocity prediction programme for the 1995 Black Magic team. In 2007 in Spain, each boat will be guided by several different maths teams, some of them from the NZIMA.

The first maths team contributes to the yacht's design. They use computational fluid dynamics programs to compute the drag forces on particular hull designs. Similar codes are used to predict the lift and drag forces on sail designs. Velocity prediction programmes (VPP) take force models from sails and hulls and integrate those in a set of equations.

Andy Philpott, top, and Hamish Shield. Photos: Godfrey Boehnke

"So if you have a sail at this angle to the wind and this wind speed you get a certain set of forces," says Philpott. "If the boat is moving through the water at this speed, then another set of forces is acting on the hull. In each case the force vector and the moment vector both have three components. At an equilibrium speed, the sum of the forces and moments has to equal zero. Once these six equations are solved, the boat's speed can be predicted before it is built."

The second maths team concentrates on structural mechanics, trying to keep the weight as low as possible while ensuring the reliability of the boat under load. In 2007, this aspect is bound to get more attention from Emirates Team New Zealand after a broken mast fitting and hull breaches undermined their last cup defence.

Another maths team builds race modelling programs. They feed the information from the VPP into a program that pits two boats with different rigs and hulls racing against each other in a variety of simulated wind conditions to see how they perform. Philpott's student David Teirney did this for the 2000 Cup defence. Says Philpott: "If you run thousands of races with random winds, one boat will win, say, 55 percent of races. That tells you how much better one design feature will be than another."

A different group again looks at wind conditions on the course, to help the weather teams decide which side of the start line they should be on and which side of the course they should take on the first leg. Each team gets the same data from weather buoys on the Valencia course. "The weather teams for all the syndicates are populated by New Zealanders," says Philpott. Some teams are using meteorological models, which usually give a single point forecast; some produce wider ensemble forecasts. "So it's a battle of several models," says Philpott. Emirates Team New Zealand's current models remain under wraps, but the maths is no less hotly contested for being hidden behind the action.

Philpott's approach is to treat the wind conditions as a stochastic (random) process that evolves over time, and calibrate it to real weather observations. Using the Valencia data from Emirates Team New Zealand, NZIMA-supported Masters student Hamish Shield has implemented a stochastic wind field model, aiming to predict what the wind will be doing at any point on the course when the boat gets there.

The model uses a complex set of equations developed by Professor Shane Henderson at Cornell University, who worked with Philpott and Teirney on the 2000 defence. It combines with Teirney's match racing simulation program, and uses an Excel spreadsheet and a macro to animate the results. Shield says the model enables a weather team to see the probability of one yacht reaching the first buoy first given a range of starting positions and wind conditions.

"The team with the best model will make the best decisions, get the better wind shifts and have a greater chance of winning," says Shield. His stochastic wind field model could be used for winds in any location, and for other applications such as wind farms. "It was definitely fun doing this; getting to work on this kind of project is a good reason to stay with maths."

See also

Philpott, A.B., Henderson, S.G., and Teirney, D.P., A simulation model for predicting yacht match-race outcomes, Operations Research, 52, 1, 1-16, 2004.

Philpott, A.B. Stochastic optimization in yacht racing, in Applications of Stochastic Programming, W. Ziemba and S. Wallace (ed.), SIAM, 2005.